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A STUDY OF PERMUTATION ADMISSIBILITY WITH COLORED PETRI NETS

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ABSTRACT

The permutations, which are routable through a multistage interconnection network without any conflict (known as the admissible permutations) play important role in determining the capability of the network. Admissibility of frequently used permutations becomes a crucial issue when we decide on best matching between parallel algorithms and architecture of parallel computers.

In this paper, we propose an innovative approach to analyze the permutation admissibility. The approach is centered upon colored Petri net modeling of multistage interconnection network with further analysis of associated model through exploiting colored place invariants.

I. INTRODUCTION

Rapid communications among processors are important for parallel computers. Permutations are frequently used communications. For example, data are permuted among processors for the next step of parallel algorithm or data are scrambled before they are stored in the parallel memory. Permutation can be viewed as a one-to-one and onto mapping of sources to desired destinations. A permutation is said to be *admissible* to MIN if conflict-free paths can be established for all input-output pairs simultaneously. Many kinds of MINs have been investigated for permutation admissibility. Over the years researchers have investigated the admissibility of linear-permutation-class (LIN) and bit-permutation-class (BPC) permutations to shuffle-type [4-6], cube-type [3,7] and hypercube [8] networks. These works are mainly centered upon use of algebraic techniques or so-called window method [7] to find out a better routing algorithm to generate particular permutation for associated networks. Deciding on whether an arbitrary permutation is admissible to a given MIN is still difficult problem.

Petri nets provide a framework for the modeling, simulation, specification and validation of dynamic systems. Petri nets are flexible specification languages for concurrent systems and offer wide range of applicability. Petri nets have been successfully employed in many scientific and industrial applications. In [1-2] CP-nets particularly have successfully been employed for analysis the permutation capability in MINs.

CP-nets are class of high level Petri nets which in turn evolved from finite state automata as specification for concurrent systems. The use of colors allows a convenient way to manipulate information that we have been able to exploit in modeling of MINs. The convenience is both in the size of the model, which becomes comparable to that of the MIN itself, and in easiness with which the behavioral features are captured.

In this work we propose a new approach to investigate the permutation admissibility, which is based on modeling of MINs with colored Petri nets (CP-nets). The main idea behind of the proposed approach is to reduce the permutation admissibility problem to the problem of reachability in CP-nets. There exist a plenty of well-established and easy-to-use methods to analyze reachability in ordinary P/T-nets including reachability graph, coverability graph and place invariants methods. On the other hand, it is broadly-known that ordinary P/T-nets and CP-nets have the same computational power meaning that given a CP-net, one can construct an equivalent P/T-net and vice versa. Once CP-net modeling of a MIN has completed we can further transform related CP-net to an equivalent P/T-net and use the method of place invariants to decide on permutation reachability in CP-net and herein decide on permutation admissibility to the given MIN.

The remainder of the paper is organized as follows. CP-net model of a MIN is presented in Section II. Colored place invariants are discussed in Section III. In Section IV colored place invariants are implemented to MINs. Finally, Section V is a conclusion.

II. MIN CP-NET MODELING

As it is defined in [1-2] a CP-net of a MIN is 9-tuple $CPN = (S, P, T, A, N, C, G, E, I)$, such that

- $\Sigma = \{INPUT, CONDITION\}$, where $INPUT = \{0,\ldots,2^n - 1\}$, $CONDITION = \{OK\}$;
- $P = P_{IN} \cup P_{AUX} \cup P_{OUT} \cup P_{CDN}$, where
  - $P_{IN} = \{IN_i\}$, $i = 1,\ldots,2^{n-1}$, is the set of *input places*;
  - $P_{AUX} = \{AUX_i\}$, $i = 1,\ldots,m2^{n-1}$, is the set of *auxiliary places*;
  - $P_{OUT} = \{OUT_i\}$, $i = 1,\ldots,2^n$, is the set of *output places*;
  - $P_{CDN} = \{CDN_i\}$, $i = 1,\ldots,(m+1)2^{n-1}$, is the set of *condition places*;
- $T = T_{SW} \cup T_{DP}$, where
  - $T_{SW} = \{SW_i\}$, $i = 1,\ldots,m2^{n-1}$, is the set of *switch transitions*;
  - $T_{DP} = \{DP_i\}$, $i = 1,\ldots,2^{n-1}$, is the set of *display transitions*;
- $A = A_{SW} \cup A_{PTN} \cup A_{CDN} \cup A_{DP}$, where
  - $A_{SW} \subseteq (P_{IN} \times T_{SW}) \cup (T_{SW} \times P_{AUX})$ is the set of *switch arcs*;
  - $A_{PTN} \subseteq (P_{AUX} \times T_{SW})$ is the set of *pattern arcs*;
  - $A_{CDN} \subseteq (P_{CDN} \times T) \cup (T \times P_{CDN})$ is the set of *condition arcs*;
  - $A_{DP} \subseteq (P_{AUX} \times T_{DP}) \cup (T_{DP} \times P_{OUT})$ is the set of *display arcs*;
- $C(p) = \begin{cases} CONDITION, & \text{if } p \in P_{CDN} \\ INPUT, & \text{otherwise;} \end{cases}$
- $G(t) = \text{true } \forall t \in T$;
- $E(a) = \begin{cases} NEXT, & \text{if } a \in A_{CDN} \\ (Var(a))_{MS}, & otherwise \end{cases}$
- where $Type(Var(a)) = INPUT$;
- $I(p) = \begin{cases} 1'(2i-2) + 1'(2i-1), & \text{if } p \in INi, i = 1,\ldots,2^{n-1} \\ (empty, & otherwise. \end{cases}$

In the above definition, the set of *places P*, the finite set of *transitions T*, the finite set of *arcs A*, and the *node function N* determine a directed graph representing the static structure of the CP-net. However, the finite set of non-empty types, called *color sets* Σ, the *guard function G*, the *colour function C*, and the *arc expression function E* determine the dynamic structure of the CP-net. *I* the initialization function is used to generate the initial marking which represents the initial state of the system. The sets P, T and A are pairwise disjoint i.e. $P \cap T = P \cap A = T \cap A = \varnothing$. The node function $N : A \to P \times T \cup T \times P$ associates each arc with a pair of nodes. Each arc connects a place p to a transition t or vice versa. The colour function $C : P \to \Sigma$ assigns a colour set $C(p)$ to each place p. The guard function G is defined from T into expressions such that

$$\forall t \in T : [\text{Type}(G(t)) = \mathbf{B} \wedge \text{Type}(\text{Var}(G(t))) \subseteq \Sigma].$$

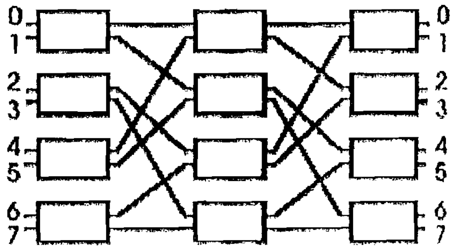

Figure 1. 8×8 **Omega network.**

As an example of CP-net diagram of 8×8 Omega network (Figure1) is illustrated in Figure2.

III. COLORED PLACE INVARIANTS

Place invariants are equations that are satisfied in all reachable markings of the CP- net and are used to verify certain properties of the CP-net. The incidence matrix I of a CP-net contains a row for each place $p \in P$ and a column for each transition $t \in T$. Each matrix element is defined by

$$I(p,t)(b) = E(t,p) < b > -E(p,t) < b >,$$

where tokens are removed from and added in places according to the expressions $E(t,p) < b >$ and $E(p,t) < b >$, respectively. In the above formula *I(p,t)(b)* describes how the marking of a place p is changed when the transition t occurs with a binding b.

The important ingredients of the place invariants are incidence matrix I, step Y and two markings M_0 and M'. An incidence matrix I is another way to represent the structure of CP-net, which turns out to be convenient when we deal with calculation of invariants. Step Y is a collection of transitions for all enabled bindings. Finally, M_0 and M' stand for the initial and the final markings. Analysis of reachability by the method of place invariants is described by the equation:

$$M' = M_0 + (I \circ Y)(p),$$

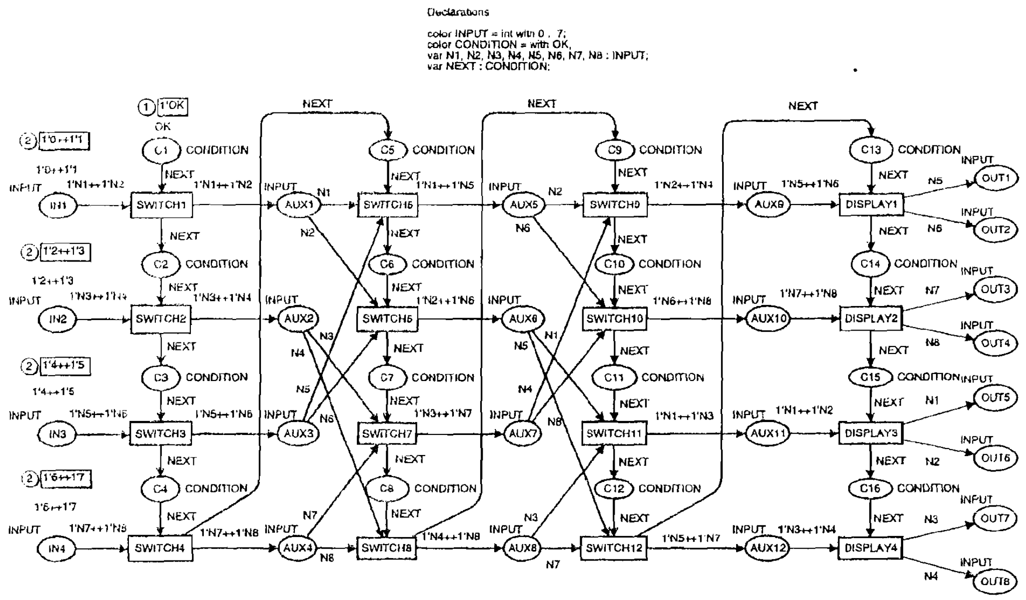

Figure 1. 8×8 **Omega network CP-net diagram.**

where

$$
\begin{aligned}
(I \circ Y) &= \sum_{t \in T} \sum_{p \in Y(t)} I(p,t) Y(t) \\
&= \sum_{t \in T} \sum_{p \in Y(t)} \sum_{b \in Y(t)} (E(t,p) < b > - E(p,t) < b >) \\
&= \sum_{(t,b) \in Y} \sum_{p \in Y(t)} E(t,p) < b > - \sum_{(t,b) \in Y} \sum_{p \in Y(t)} E(p,t) < b >
\end{aligned}
$$

.

IV. CASE STUDY

As an example, let us consider the incidence matrix for 8×8 Omega network. The elements of related incidence matrix are linear functions:

$$
\begin{aligned}
&Id(1'n_i + +1'n_{i+1}) = 1'n_i + +1'n_{i+1}, \\
&Fork(1'n_i + +1'n_{i+1}) = 1'n_k, k \in \{i, i+1\} \\
&Join(1'n_i, 1'n_j) = 1'n_i + +1'n_j, n_i \neq n_j \\
&Cdn(1'n_i + 1'n_{i+1}) = 1'OK,
\end{aligned}
$$

where $n_i, n_{i+1}, n_j, n_k \in C(p)$, $1 \leq i, j, k \leq 8$. The identity function Id is used to indicate the binding elements (transitions together with the surrounding arcs and arc expressions), which move tokens from source places into destination places and keep the number and type of the tokens as they are. These binding elements appear in the left side of CP-net diagram of Figure 2. The binding elements in the middle of CP-net diagram (see Figure 2) are determined by the function $Fork$. The function $Join$ represents binding elements that are arranged on the right side in Figure 2. Finally, the binding elements between conditional places and transitions (vertical arcs) are represented by the function Cdn. Zero function is denoted by an empty matrix entrance. Corresponding incidence matrix is shown in Table 1. In order to increase the readability, we list the places together with their color sets at the left of the table and transitions together associated bindings at the top.

In order to perform fully automatic calculation of colored place invariants we need to write down all place invariants, each representing a linear equation, represent CP-net as incidence matrix, and solve a homogeneous matrix equation. If the matrix equation has a solution (unique or multiple) then marking M' is reachable from the initial marking, i.e. $M' \in R(M_0)$. The non-existence of the solution indicates that M' is not reachable from the initial marking.

Table 1. 8×8 **Incidence matrix of Omega network CP-net.**

		INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT
		SW1	SW2	SW3	SW4	SW5	SW6	SW7	SW8	SW9	SW10	SW11	SW12	DISPLAY1	DISPLAY2	DISPLAY3	DISPLAY4
INPUT	IN1	-Id															
INPUT	IN2		-Id														
INPUT	IN3			-Id													
INPUT	IN4				-Id												
INPUT	AUX1	Id				-Fork	-Fork										
INPUT	AUX2		Id					-Fork	-Fork								
INPUT	AUX3			Id		-Fork	-Fork										
INPUT	AUX4				Id			-Fork	-Fork								
INPUT	AUX5					Join				-Fork	-Fork						
INPUT	AUX6						Join					-Fork	-Fork				
INPUT	AUX7							Join		-Fork	-Fork						
INPUT	AUX8								Join			-Fork	-Fork				
INPUT	AUX9									Join				-Id			
INPUT	AUX10										Join				-Id		
INPUT	AUX11											Join				-Id	
INPUT	AUX12												Join				-Id
INPUT	OUT1													Fork			
INPUT	OUT2													Fork			
INPUT	OUT3														Fork		
INPUT	OUT4														Fork		
INPUT	OUT5															Fork	
INPUT	OUT6															Fork	
INPUT	OUT7																Fork
INPUT	OUT8																Fork
CONDITION	C1	-Cdn															
CONDITION	C2	Cdn	-Cdn														
CONDITION	C3		Cdn	-Cdn													
CONDITION	C4			Cdn	-Cdn												
CONDITION	C5				Cdn	-Cdn											
CONDITION	C6					Cdn	-Cdn										
CONDITION	C7						Cdn	-Cdn									
CONDITION	C8							Cdn	-Cdn								
CONDITION	C9								Cdn	-Cdn							
CONDITION	C10									Cdn	-Cdn						
CONDITION	C11										Cdn	-Cdn					
CONDITION	C12											Cdn	-Cdn				
CONDITION	C13												Cdn	-Cdn			
CONDITION	C14													Cdn	-Cdn		
CONDITION	C15														Cdn	-Cdn	
CONDITION	C16															Cdn	-Cdn

A problem causes when we try to employ computer tools for automatic calculation of place invariants. Since the elements of incidence matrix are functions rather that numeric values, the method of colored place invariants is less useful for computerized analysis. Instead we would suggest CP-net unfolding and then verification of reachability in terms of place invariants for ordinary P/T-nets. Consequently an approach proposed in this work

V. CONCLUSION

In this paper we introduce a novel technique for analysis of permutation admissibility to a MIN. Based on this technique we can decide on permutation admissibility through analysis of reachability property of related CP-nets. The procedure passes through the following stages:

- CP-net modeling of a MIN
- Computing colored place invariants
- Computing CP-net unfolding
- Computing place invariants
- Reachability analysis through matrix equations.

The main advantage of the proposed technique is that it allows us to perform fully automatic analysis of the objects.

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